

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

	$w_{0.005}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$\frac{n(n+1)}{2}$
$n = 4$	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	66	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	83	91	98	105	210
21	44	50	59	68	78	91	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	248	266	282	297.5	595
35	160	175	196	214	236	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	266	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n+1)/4] + z_p \sqrt{n(n+1)(2n+1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE: Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

*The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^+ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T^+ < w_p) \leq p$ and $P(T^+ > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^+ less than (or greater than) but not including the appropriate quantile.

Critical Values of the Mann-Whitney-Wilcoxon Test Statistic

This table gives the values of u_α for which $P\{U > u_\alpha\} \leq \alpha$ for some selected values of m , n , and α . Critical values in the lower tail may be obtained by symmetry from the equation $u_{1-\alpha} = mn - u_\alpha$.

m	α	n								
		2	3	4	5	6	7	8	9	10
2	.01	4	6	8	10	12	14	16	18	20
	.025	4	6	8	10	12	14	15	17	19
	.05	4	6	8	9	11	13	14	16	18
	.10	4	5	7	8	10	12	13	15	16
3	.01		9	12	15	18	20	20	25	28
	.025		9	12	14	16	19	21	24	26
	.05		8	11	13	15	18	20	22	25
	.10		7	10	12	14	16	18	21	23
4	.01			16	19	22	26	29	32	36
	.025			15	18	21	24	27	31	34
	.05			14	17	20	23	26	29	32
	.10			12	15	18	21	24	26	29
5	.01				23	27	31	35	39	43
	.025				22	26	29	33	37	41
	.05				20	24	28	31	35	38
	.10				19	22	26	29	32	36
6	.01					32	37	41	46	51
	.025					30	35	39	43	48
	.05					28	33	37	41	45
	.10					26	30	34	38	42
7	.01						42	48	53	58
	.025						40	45	50	55
	.05						37	42	47	52
	.10						35	39	44	48
8	.01							54	60	66
	.025							50	56	62
	.05							48	53	59
	.10							44	49	55
9	.01								66	73
	.025								63	69
	.05								59	65
	.10								55	61
10	.01									80
	.025									76
	.05									72
	.10									67

Source. Adapted by permission from Table 1 of L. R. Verdooren, Extended tables of critical values for Wilcoxon's test statistic, *Biometrika* 50 (1963), 177-186, with the kind permission of Professor E. S. Pearson, the author, and the *Biometrika* Trustees.

Table 1. QUANTILES OF THE KRUSKAL-WALLIS TEST STATISTIC FOR SMALL SAMPLE SIZES*

Sample Sizes	$W_{0.10}$	$W_{0.05}$	$W_{0.01}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.773	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

SOURCE. Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.

* The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.